

## Simulation of Low-Cost Flights to Asteroids from the Vicinity of the Sun-Earth Libration Points

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### Abstract

The work is devoted to the study of the motion of the spacecraft in orbits in the vicinity of the L<sub>1</sub> and L<sub>2</sub> Sun-Earth libration points, as well as associated invariant manifolds and homoclinic and heteroclinic connections, in order to flyby near asteroids approaching the Earth. The existence of stable and unstable manifolds opens up the possibility of redirecting a spacecraft from a bounded orbit almost without fuel costs to the area where a near-Earth object will pass. The main focus is on trajectories where the spacecraft initially approaches the celestial body under study and then returns to the vicinity of the libration point at which it originally functioned. The Spectrum-Roentgen-Gamma space observatory acts as a spacecraft for which a similar concept of an asteroid exploration mission is proposed. The potentially hazardous asteroid Apophis has been selected as the target celestial body, which will approach the Earth in April 2029. By this time, the observatory should have completed its target mission and will be able to be redirected to the Apophis asteroid. Preliminary calculations show that there are a number of possible scenarios for such a transfer that satisfy the existing restrictions on the magnitude of the characteristic velocity.

**Keywords:** Libration Points, Invariant Manifolds, Spectrum-Roentgen-Gamma, Apophis, Asteroid Flyby

### Nomenclature

$\Delta V$  – the minimum required impulse value,

$V_{rel}$  – the relative velocity of the asteroid flyby at the moment of the close approach.

### Acronyms/Abbreviations

Spectrum-Roentgen-Gamma (SRG),

General Mission Analysis Tool (GMAT).

## 1. Introduction

The L<sub>1</sub> and L<sub>2</sub> Sun-Earth libration points are widely used in space missions. During practical implementation, the spacecraft does not operate at the libration point itself, but in a so-called bounded orbit near it. Without corrections, the spacecraft cannot stay in such an orbit for a long time, since the L<sub>1</sub> and L<sub>2</sub> libration points are points of unstable equilibrium. In this case, the spacecraft will move along a trajectory from the set of an unstable manifold and will leave the vicinity of the libration point. There is also an opposite set of trajectories, along which the spacecraft enters a bounded orbit. It is called a stable manifold. Both sets form a single set, an invariant manifold.

Among the trajectories of the invariant manifold, there are those along which the spacecraft first departs from its initial orbit and then returns to it. Such trajectories are called homoclinic. If the spacecraft returns to another orbit, not necessarily located in the vicinity of the same libration point, then in this case we are talking about heteroclinic trajectories. The advantage of the second type of trajectories is the ability to move between the L<sub>1</sub> and L<sub>2</sub> Sun-Earth libration points, the distance between which is about three million kilometers, with almost no fuel consumption. Thus, libration points may also be of interest for missions to study near-Earth asteroids [1], since the trajectories of these celestial bodies may pass close to invariant manifolds.

## 2. Material and methods

The orbit of the Spectrum-Roentgen-Gamma Space Observatory was chosen as the initial one [2]. This spacecraft is currently operating in the vicinity of the L<sub>2</sub> Sun-Earth libration point, and its main mission should be completed in 2026. It is assumed that at the end of all the tasks of the project, the SRG observatory will have enough characteristic velocity to be transferred to the trajectory of an invariant manifold followed by a flyby of an asteroid approaching the Earth.

The Apophis asteroid was chosen as the celestial body to which transfer trajectories of the SRG spacecraft were calculated. This celestial body is notable for the fact that its next close approach to the Earth will take place on April 13, 2029 at a distance of approximately 30000 kilometers from its surface.

To simulate the motion of the SRG spacecraft and the Apophis asteroid, the NASA GMAT software package was used [3], which allows numerical integration of the differential equations of motion by the 8-th order Runge-Kutta method in the JPL DE405 ephemeris model. All trajectories are represented in a rotating coordinate system centered at the  $L_2$  libration point of the Sun-Earth system. Figure 1 shows several revolutions of the simulated SRG orbit in the period from 2026 to 2029 and a part of the Apophis trajectory, which falls on April 2029.

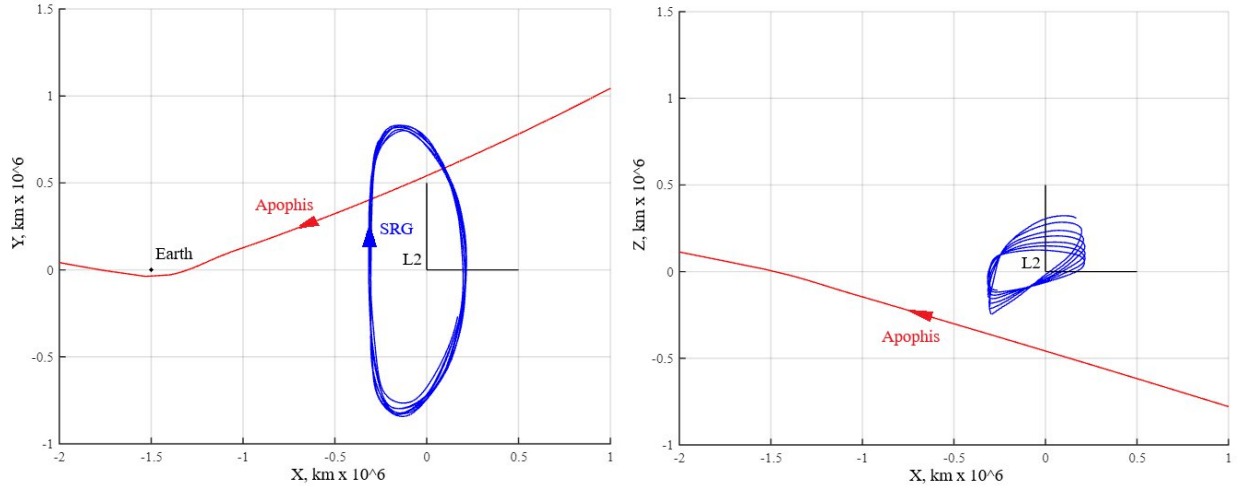


Fig. 1. Trajectories of the SRG Observatory and the Apophis asteroid

### 3. Theory and calculation

In this paper, we propose two options for redirecting the SRG observatory to the Apophis asteroid. In the first case, after applying an impulse along the escape direction [4], the spacecraft redirects to the trajectory of an invariant manifold associated with the initial orbit in the vicinity of the  $L_2$  Sun-Earth libration point, and after some time passes near the asteroid. In the second scheme, the observatory first moves along a heteroclinic trajectory to an orbit near the  $L_1$  Sun-Earth libration point, and then approaches Apophis along the trajectory of an unstable manifold of a new bounded orbit.

To construct these transfer schemes, it is necessary to calculate the so-called tube of an unstable invariant manifold. The date and the magnitude of the required impulse implementation corresponds to one of the tube trajectories (see Fig. 2).

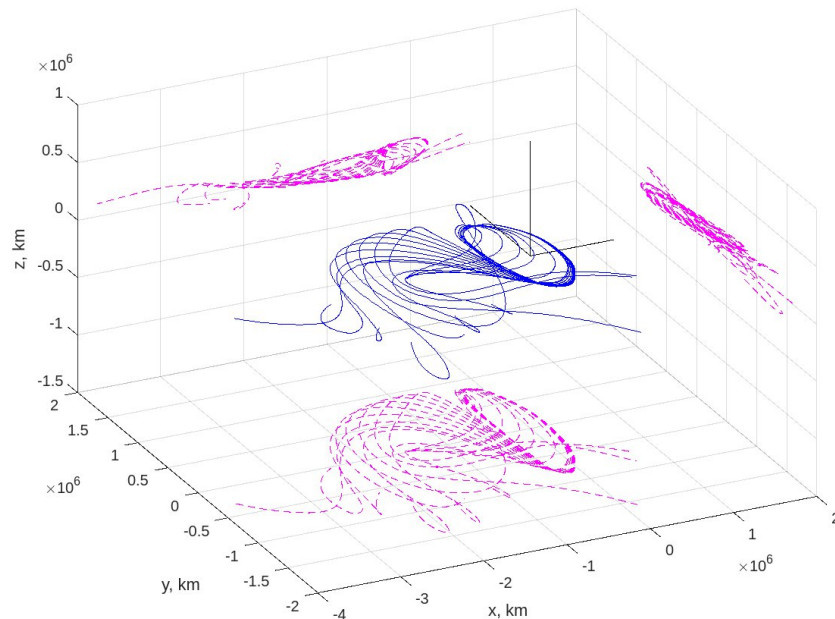


Fig. 2. An example of an unstable manifold

## 4. Results

### 4.1 The first transfer scheme

To design the first transfer scheme, we need to determine the trajectory from the set of an unstable manifold along which the minimum distance is reached during the approach to the asteroid. Also, we considered only those trajectories, when moving along which the SRG spacecraft does not leave the near-Earth space and does not collide with the Earth or the Moon. Among them, a number of trajectories were found that ensure an approach to Apophis. For all such trajectories, the encounter date is April 13, 2029.

For a closer approach to the Apophis asteroid, it is necessary to change the inclination of the SRG geocentric orbit. The scheme of such a flight is shown in Fig. 3. The trajectory of the SRG space observatory consists of several sections: first, the spacecraft is transferred to a trajectory from the unstable manifold after applying  $\Delta V_1 = 0.5$  m/s on August 28, 2028 (the trajectory before the impulse implementation is shown in blue, the ones after is shown in green). Then,  $\Delta V_2 = 35$  m/s on January 17, 2029 is applied, and the SRG spacecraft is directed to the Apophis asteroid (the trajectory after the impulse implementation is shown in purple). In this scenario,  $V_{rel} = 6.8$  km/s and this value is almost one and a half times less than in the case of the passage of the Mathilde asteroid by the interplanetary station “NEAR Shoemaker” in June 1997 [5].

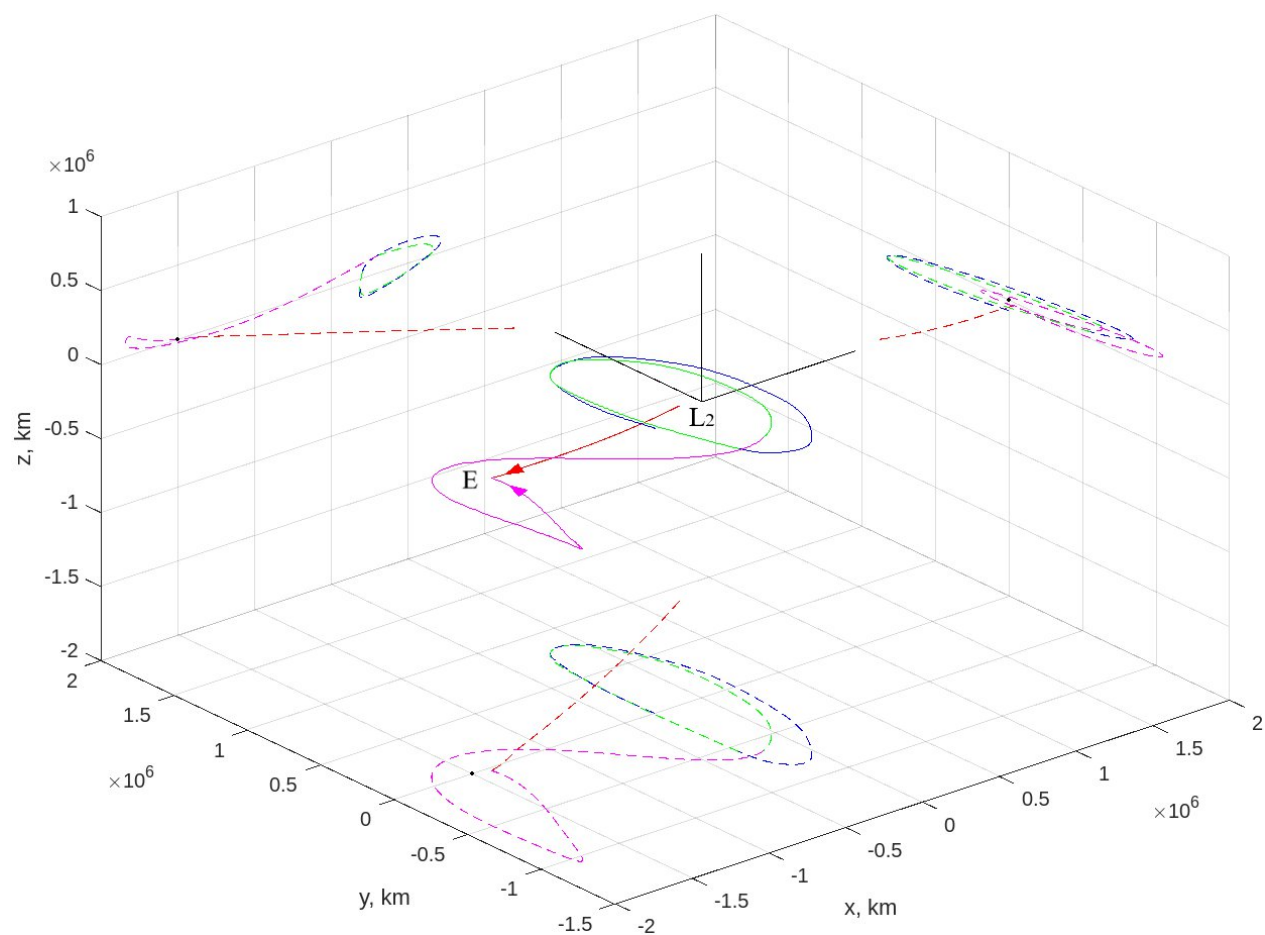


Fig. 3. Transfer trajectory of the SRG spacecraft to a close approach to Apophis

After the close approach to the Apophis asteroid, it is possible to return the SRG observatory to a bounded orbit in a vicinity of the  $L_2$  Sun-Earth libration point (see Fig. 4). To solve this problem, it is necessary to apply an impulse  $\Delta V_3 = 6.1$  m/s in the pericenter of the geocentric orbit in order to transfer the SRG spacecraft to a bounded orbit around the  $L_1$  Sun-Earth libration point (this part of the trajectory is shown in yellow). Then, the spacecraft can be redirected to a heteroclinic trajectory ( $\Delta V_4 = 0.01$  m/s) from an orbit around  $L_1$  to an orbit around  $L_2$ .

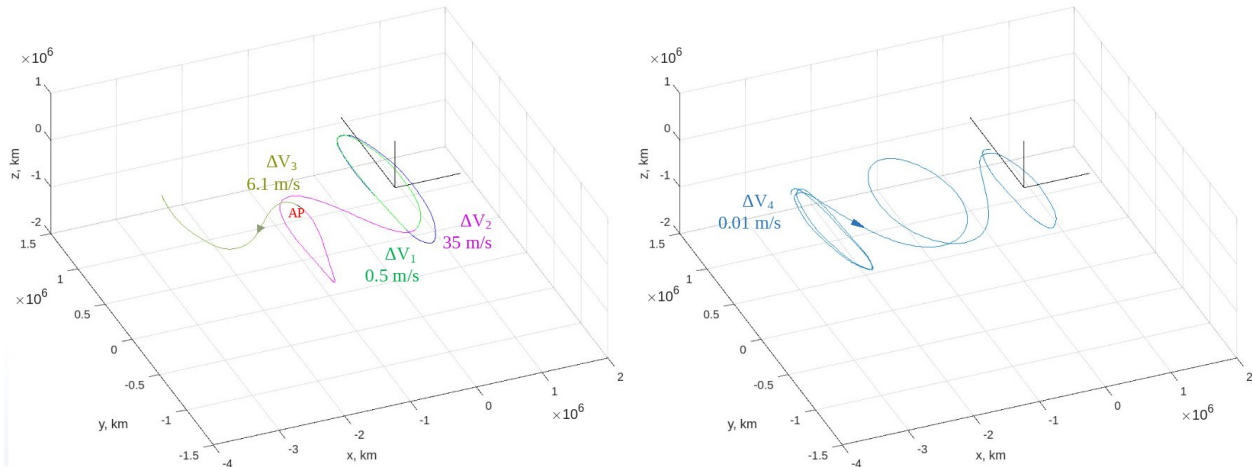


Fig. 4. The full scenario №1 of the SRG-Apophis flight

#### 4.2 The second transfer scheme

In the second scenario it is necessary to transfer the SRG spacecraft to the trajectory of the heteroclinic connection between the initial orbit near the  $L_2$  Sun-Earth libration point and the new one in the vicinity of the  $L_1$  libration point of the same system (see Fig. 5). In this case,  $\Delta V_1 = 0.1$  m/s and the arrival of the spacecraft into a new orbit takes place almost 9 months after impulse implementation on August 24, 2027.

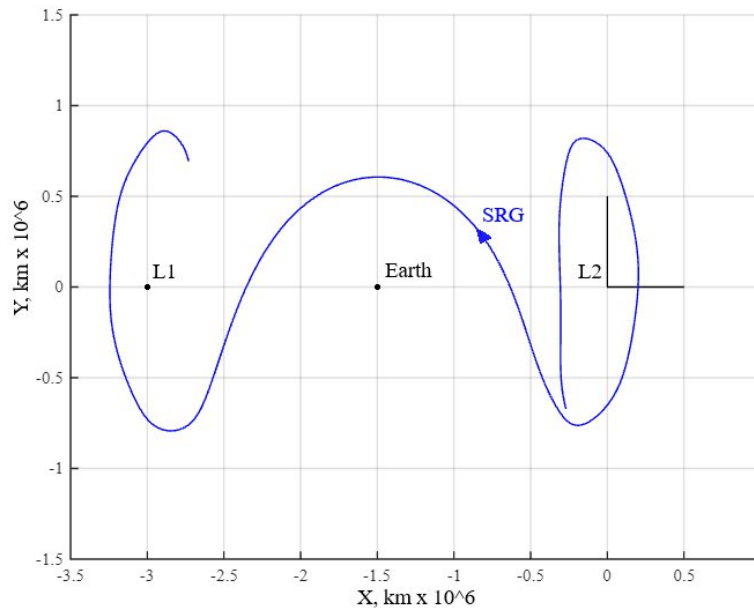


Fig. 5. Heteroclinic connection between orbits around the  $L_1$  and  $L_2$  Sun-Earth libration points

The next impulse, aimed to transfer the spacecraft on a trajectory of approach to the Apophis asteroid, is calculated in the same way as the first scheme. The trajectory of the SRG observatory after the implementation  $\Delta V_2 = 1$  m/s on September 5, 2028 is shown in Figure 6 in blue. In this case, the approach to Apophis falls on April 14, 2029. The magnitude of  $V_{rel}$  is the same as in the first scheme. The disadvantage of this scenario is that after the asteroid flyby, the observatory must be decelerated at the pericenter in order to stay near the  $L_2$  libration point. The cost of such a maneuver can be almost 30 m/s.

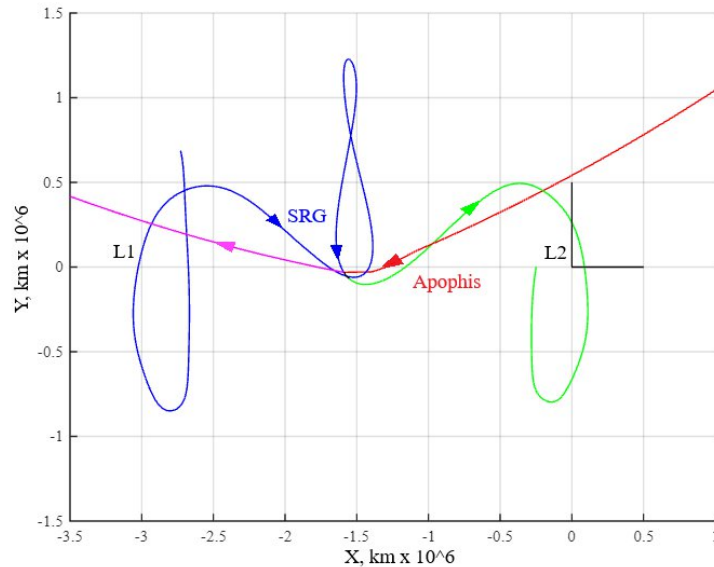


Fig. 6. Transfer of the SRG spacecraft from a new orbit near the  $L_1$  Sun-Earth libration point to Apophis

## 5. Discussion

This concept for designing such missions can be applied to other spacecraft orbiting near libration points. For example, James Webb Space Telescope, Euclid, Gaia, etc. It is also worth noting that not only Apophis will move near the Sun-Earth libration point. Another potentially hazardous asteroid, 1997 XF<sub>11</sub>, may also be of interest.

## 6. Conclusions

The paper presents the results of simulating two transfers of the SRG observatory to the Apophis asteroid along the trajectories from the set of an invariant manifold. It is shown that in both cases this celestial body can be reached with minimal fuel consumption. The proposed concept may be useful for future missions at the Sun-Earth libration points, especially for small spacecraft due to the low fuel costs for the implementation of such flights.

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